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Article

A Study of Properties and The Formation of Shapes in Classical Geometry and Chaos Theory

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Abstract: The study examines the mathematical properties and mechanisms that give rise to patterns at the intersection of traditional geometry and random theory. It discusses how ordered geometric systems characterised by stability transform into chaotic and complex patterns under the influence of non-linear dynamical equations, as well as relying on computer modelling and the analysis of mechanisms and data trajectories. The results showed that even minor changes in the initial conditions can radically reshape these geometric structures, potentially resulting in fractal patterns characterised by infinite self-similarity, This analysis demonstrates that the apparent chaos in chaotic systems follows an internal geometric order of infinite precision governed by attractive forces. The results contribute to supporting the theoretical understanding of complex dynamical systems across various fields of theoretical computer science, whilst also opening up new horizons for the application of these geometric patterns in practical fields such as cryptography, computer simulation, and the analysis of complex physical and environmental systems

Keywords: Fractal Patterns, Characterized, Chaos Theory, Cryptography, Sierpinski

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1. Introduction

When the theory of chaos had emerged and gained understanding during the 1960s, difference equations had produced some intriguing pictures. During the 1970s, those pictures were called "fractals," thus forming a field of study that has intrigued many mathematicians and scientists. Fractals can be formed using various techniques, but what do they represent? It is an image which is said to be self-similar [1-3]. A self-similar image is defined as an image where itself repeats at a reducing scale. Thus, there would be smaller images of itself inside the larger one. If one zooms into a specific part of a fractal in a particular ratio, one will end up seeing the same image again. This can be repeated infinitely as the process of zooming in could go on and on, Such a definition of self-similarity is rather general because a line segment is self-similar, but a line segment is not a fractal. Defining the term “fractal” precisely is not an easy task to accomplish; consequently, there is no general definition of a fractal because there is no single definition that captures everything about a fractal. It is important to remember that chaos also lacks a clear definition. Because fractals are produced from chaos, it follows that it is impossible to have a definitive definition of fractals. Although defining a fractal is not an easy task, Although we cannot provide an exact definition of the fractal, we can still state some characteristics that all fractals should have. In order to be called a fractal, a fractal should satisfy the following criteria [4-9]:

1. A fractal should be infinitely detailed, i.e. it has details at any scale.
2. It cannot be described using classic geometrical methods, since its shapes are too irregular.
3. Most fractals are self-similar, although there are exceptions.

These fractals were unwittingly discovered and constructed by mathematicians in history. The reason why it took a while for fractals to be studied in detail is because they require technology to be analyzed. Using the concepts of chaos theory, fractals can be formed through dierence equations. To get a clearer depiction of how the orbits behave, a lot of iterations for a number of points have to be computed [10]. The creation of such images was a result of entering data into a computer program. This program would allow the program to generate graphs depicting the orbits of the points faster than a person could do manually. With computers becoming increasingly common in the 1960s, this is the reason why the study of chaos theory was also being developed in this period. As mathematicians began studying chaos theory by inputting data and generating graphs through computers, they found images of strange attractors and fractals [11].

2. Materials and Methods

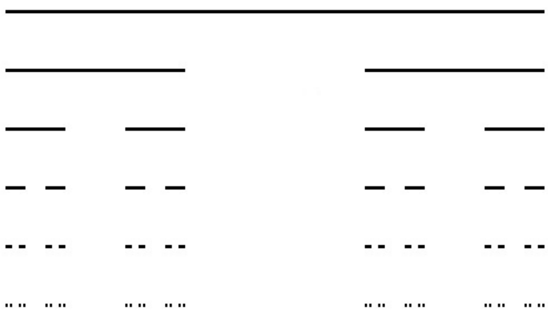
This study employed a theoretical and computational modelling approach to examine the formation and properties of geometric patterns in classical geometry and chaos theory. The analysis focused on representative fractal structures, including the Cantor set, Sierpinski triangle, Koch curve, and Koch snowflake, which were constructed through repeated geometric transformations from an initial set to successive iteration stages. Each structure was evaluated in terms of self-similarity, scaling ratio, dimensional characteristics, perimeter, area, and behaviour as the number of iterations approached infinity. To investigate chaotic behaviour, the logistic map was analysed through iterative calculations using different parameter values and initial conditions. The resulting numerical trajectories and graphical patterns were compared to identify fixed points, periodic behaviour, bifurcation, sensitivity to initial conditions, and the emergence of attractors. The findings from the geometric constructions and computational iterations were subsequently interpreted comparatively to explain how simple deterministic rules can generate stable, fractal, and apparently chaotic patterns while remaining governed by precise mathematical relationships.

3. Results and Discussion

The Theory of Fractals

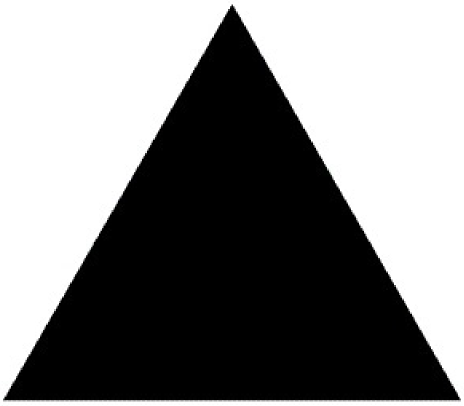
Some examples and constructions will help us comprehend the characteristics of fractals in order to give a clear understanding of what a fractal is. A base set and the transformation that will be performed to it are required in order to create a fractal. The initiator is the base set E_0. The generator is the first transformation of the set E_1, and En is the nth transformation of the set. The $\lim_{n\to\infty} E_n$ of E_n as n approaches infinity is the fractal F. Some examples are given below[12].

Example 1” The Cantor set is among the most well-known instances of a fractal. Long before the word "fractal" was even used, George Cantor explored it in the late 1800s. As an exercise for this example, build the Cantor set using a line segment of length 1. The initiator, E_0, is this line. Next, we remove the center third of this line, leaving us with two 1/3-length lines.This set will be referred to as the generator, or E_1. Now remove the center third from both lines once again to obtain four lines of length 1/9 each. We will refer to this group of lines as E_2. Continue doing this indefinitely. The resultant set En will then include 2n lines of length 3^{-n}. The Cantor set is given by the limit. (See the figure). This set is completely unconnected and self-similar [13].

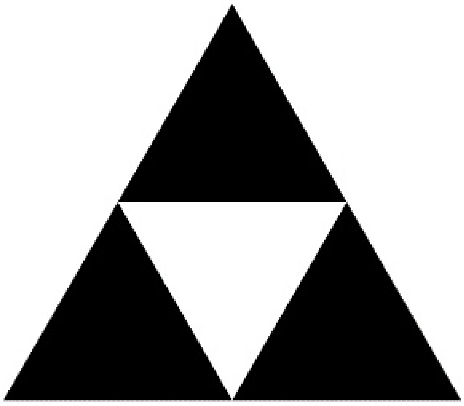


Example 2" The Sierpinski Triangle (Sierpinski gasket) is yet another well-known fractal. The Sierpinski triangle was first presented by Polish mathematician Waclaw Sierpinski in 1906.

An equilateral triangle that is completely filled with its side equal to 1 serves as the first figure in order to make the Sierpinski triangle. By connecting the midpoints of each side 4 times, we divide the initial triangle into four small triangles. We remove the central triangle to get three triangles, which are one fourth of the initial triangle's area. This gives us the first generator E_1. Repeat this process on each of the triangles and remove the middle triangle, thereby creating 9 triangles. Thus, E_n will consist of 3^n triangles with area 4^{-n}. As we take the limit $n \to \infty$ we get the Sierpinski triangle[14].

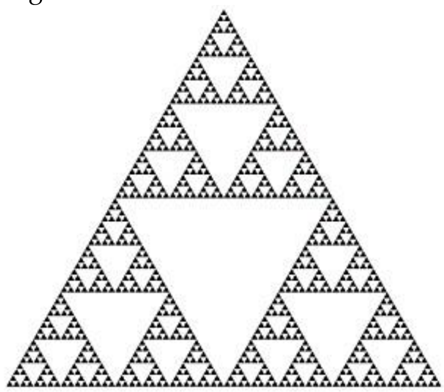


Stage E_0



Stage E_1

Stage E_2



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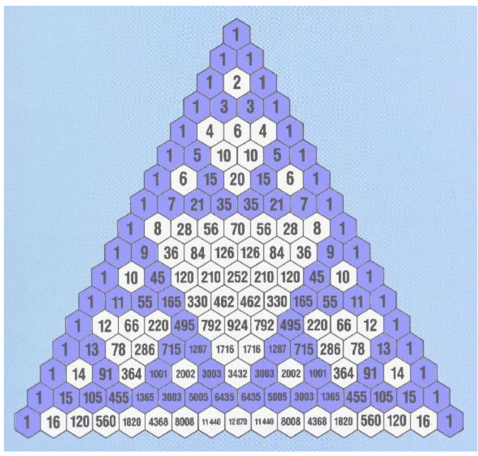
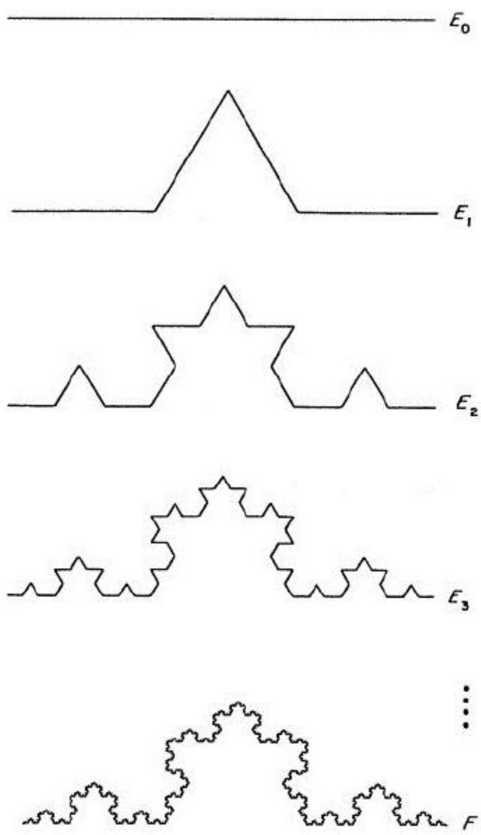
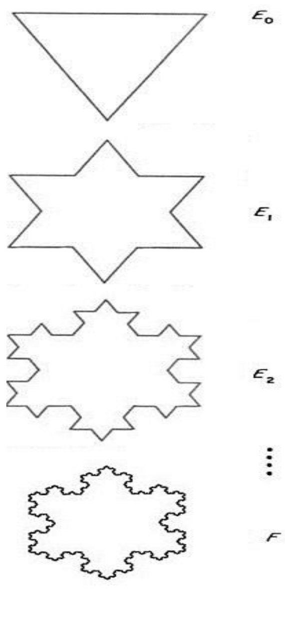
Another observation that could be considered interesting is the fact that the Sierpinski triangle can be compared to Pascal's triangle in relation to the modulo two function. If we take a look at Pascal's triangle modulo two, it becomes clear that the elements that have the value 1 form Sierpinski's triangle, whereas the filled triangles are made up of the parts with the value 0. All odd numbers are shaded in the figure below, whereas all even numbers are not. As a result, odd numbers are equivalent to 1 modulo two and even numbers to 0 modulo two.



Example :3" The Koch curve is the invention of From 1904, Helge von Koch. With the exception of the Koch curve's non-one-dimensionality, the generation of the Koch curve is nearly identical to that of the Cantor set. The straight line, such as a line of length 1, is the initiator of the Koch curve. In the generator stage, the center third of the straight line is removed and replaced with an equilateral triangle of side $\frac{1}{3}$ without any foundation at all. Next, take off the middle third of each line segment and put an equilateral triangle in its place. Interestingly, the length of the initiator is 4/3 for E_1, 16/9 = (4/3)^2 for E_2, and so on. Therefore, the curve's length for the nth iteration is (4/3)^n. Next is the fractal itself, whose length is limitless since n →∞. It is among the most significant characteristics of fractals that occur in dimensions greater than the first [16].



There have been instances where the Koch curve has been altered to form other mathematical curves. The Koch Snowflake is among them. Instead of a line, the Koch Snowflake's initiator is an equilateral triangle. The Koch Snowflake may be formed by using the same transformation technique to each side of the triangle as was used to produce the Koch curve. Because each side of the triangle has an unlimited length but the area is finite, the triangle's perimeter becomes limitless.



In other words, when we talk about the infinity of perimeter of a fractal, we mean that the object is infinitely detailed. Whenever we look at a certain portion of the object, we can find some detail or some jaggedness in it, however much we may zoom in to that particular portion. This is what Mandelbrot tried to explain when he posed the question, "How long is the coastline of Britain?”, Such fractals have already been discussed above from the geometrical point of view. There are other types of fractals that cannot be built geometrically, though. Such fractals come out of chaos. And here is one example of such complex fractals - the bifurcation diagram. The bifurcation diagram is just a graph of a chaotic function[17].

So let's consider the differenceequation of the logistic map, definedas $f_{\mu}(x) = \mu x(1-x)$, where x∈[0, 1] and $\mu \in$ (0, 4], Note also that $f\,(x) = \mu - 2\mu x$ .The goal is to determine how many periodic points a certainμ will have, and to determine the stability of each periodic point. To findthe fixedpoints, set the equation equal to x and solve. So, solve,$\mu x\,(1-x) = x\ for\ x$,This gives us two fixedpoints: $x_1^* = 0\ and\ x_2^* = \frac{\mu-1}{\mu}$, However, regardless of the value of μ we pick, these are always going to be the two fixed points, however, stability matters because the fixed point of x*=0is only stable when 0<μ<1, after deriving f at x=0, the fixed point x *is stable when 1<μ<3, but what about periodic points?

Data Interpretation & Analysis

The results of theoretical simulations and computer simulations reveal a clear evolution as we movefrom classical geometric systems to complex chaotic systems, as the evolution of the patterns under the influence of non-linear equations reveals a high sensitivity to initial conditions; any change in the initial values causes a deviation in the final pattern, and this can be mathematically demonstrated to indicate the system’s transition to random behavior. Furthermore, the results of the geometric analysis of the final shapes reveal the presence of fractal forms and structures characterised by infinite self-similarity at various levels of magnification; this fundamentally links the laws of classical geometry with chaos theory, which arises from simple equations. Nevertheless, the graphs and trajectories of the points show that this chaos is not absolute, but rather is governed by specific attractors that direct the geometry of the random system and keep it confined within precise mathematical boundaries that can be accurately modelled and interpreted.

4. Conclusion

This study addressed the following questions: the transition of stable geometric systems to chaotic systems with the aid of non-linear dynamical equations. Computational modelling techniques and the analysis of data mechanisms and trajectories were also employed in this research. It was found that the slightest change in the initial conditions can lead to the complete transformation of geometric systems into fractals exhibiting infinite self-similarity. Consequently, the analysis reveals the existence of a geometric system of infinite precision within chaotic systems, determined by the strength of the attraction. Consequently, the findings will be useful for advancing theoretical knowledge of complex dynamical systems in various fields of theoretical computer science. At the same time, these results will help to identify new possibilities for the practical application of geometric patterns in cryptography and computer simulation, amongst other areas.

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