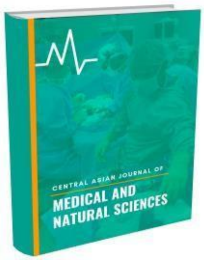

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On Mixed Fuzzy Top. R-Module

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Abstract: In this paper, we induce fuzziness of mixed top. R-module, nbhds system of mixed top. R-module and top. sub mixed top. R-module. Also, we study fuzziness of quotient mixed top. R-module.

Keywords: *: Fuzziness Top. R-Module, Fuzziness Of Mixed Top. R-Module, Fuzziness Nbhd Module, Fuzziness Sub Mixed Top. R-Module, Fuzziness Quotient Mixed Top. R-Module.*

Introduction

The fuzziness given by(Zadeh, 1965). the fuzziness of top. space studied by(Chang, 1968). the fuzziness of top. ring looked for by (Ray & Chettri, 2009) some results of fuzziness of top. ring studied by(Melgat & AL-Khafaji, 2019). fuzziness of top. module and fuzzy top. submodule given by(Al-Shamiri, 2020, Melgat2023 and Melgat 2024). Similarly, to fuzziness of bi- top. ring space we define fuzziness of bi- top. R-module. We apply the results on fuzziness of nbhd systems in a fuzziness of top. R-module grow so far, to form a fuzziness of mixed top. R-module., sub mixed top. R-module and fuzziness of quotient mixed top. R-module

For rich the work, some elements of fuzziness are given. The symbol J will denote to [0,1]. Let the set $M \neq \emptyset$:

Def. 1.1.1 (Zadeh, 1965)
the map $\partial: M \rightarrow J$ ($J = \{x: 0 \leq x \leq 1\}$) is a fz. set F in M.The belonging of $m \in F$ is $F(m)$ or m_α .

Def. 1.1.2 (Chang, 1968)

(M, β) is a fz. top. space if the class $\beta \in J^M$ is a fz. top. and the following justify:

1) $1_\emptyset , 1_M \in \beta$
2) $\forall F_1 , F_2 \in \beta \rightarrow F_1 \wedge F_2 \in \beta$
3) $\forall (F_i)_{i\in I} \in \beta \rightarrow \vee_{i\in I} F_i \in \beta$

Definition1.1.3 (Ray & Chettri, 2009)
(M, β) is a fz. top. ring space. Let $U, V \in J^M$. Then
1) $(U+V)(k) = \sup_{k=k_1+k_2} min\{U(k_1), V(k_2)\}$
2) $-V(k) = V(-k)$

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3) $(U.V)(k) = \sup_{k=k_1+k_2} min\{U(k_1), V(k_2)\}$

Def. 1.1.4 (Al-Shamiri, 2020)
Assume R is a fz. top. ring then M is a left fz. top. R-module if:
(1) M left fz. module on R.
(2) (M, β) is a fz. top. group on M and the function $l: R \times M \longrightarrow M$, $l(r, m) = r.m$, $(r \in R \; and \; m \in M)$ is a fz. conts.
Def. 1.1.5 (Al-Shamiri, 2020)
Assume (M, β) is a left fz. top. R-module space. let $F \in J^M$ and $\forall \; m, n \in M \; and \; r \in R$:
(1) $F(m+n) \geq min\{F(m), E(n)\}$.
(2) $F(m) = F(m^{-1})$.
(3) $F(rm) \geq F(m)$.
(4) $F(0) = 1$.

Def. 1.1.6 (Palaniappan N.2002)

A fz. set $F \in J^M$ is a fz. nbhd of $m \in M \Leftrightarrow \exists U \in \beta$ s.t $\alpha \leq U(m) \leq E(m)$. U is a fuzzy open set iff it's a fz. nbhd of each fz. point of its.
Theorem 1.1.7 (Melgat, 2019)
let (M, β) be a fz. top-R-module and let $\{U_0\}$ be a fund. system of fz. nbhd of 0 s.t $U(0)$ = maximum $\{M(n)\}$, $\forall n \in Ms\}$ of fz. top-R-module, then the following are justified
1) $\forall U \in U_0 \; \exists \; V \in U_0$ s.t $-V \leq U$
2) $\forall U \in U_0 \; \exists \; V \in U_0$ s.t $V + V \leq U$.
3) $\forall U \in U_0$ and $V \in U_0 \; \exists \; W \in U_0$ s.t $W \leq V \cap U$.
4) $\forall \; U \in U_0 \; \exists \; V \in U_0$ and $W_0 \in J^R$ s.t $W.V \leq U$.
5) $\forall U \in U_0$ and $r \in R \; \exists \; V \in U_0$ s.t $r.V \leq U$.
6) $\forall \; U \in U_0$ and $m \in M$, $\exists V_0 \in J^R$s.t $V.m \leq U$.

Def. 1.1.8 (Melgat & AL-Khafaji, 2020)

Let (M, β) and (M, ρ) be two fz. Top. ring spaces and let $\beta(\rho) = \{U \in I^M : \exists \; E \in \rho \; s.t \; cl_\beta(E) \leq U\}$. Then $\beta(\rho)$ is a mixed fz. Top. ring on M

Materials and Methods

The current research will be done by utilized to constructive manner, depending on mathematical definitions, constructs and proofs to derive theoretical results.

Fuzzy topology and fuzzy modules will used to constructs and definition of mixed fuzzy topological R-modules. The properties of mixed fuzzy topological R-modules will be studied, including the existence of fundamental systems of fuzzy neighborhoods, fuzzy continuity, and fuzziness of quotient modules.

The concept of bi-fuzzy topological R-modules will be introduced and studied, including the properties of fuzzy topology and fuzzy modules. The fuzziness of quotient mixed fuzzy topological R-modules will be investigated, including the properties of fuzzy topology and fuzzy modules. Examples and counterexamples will be provided to illustrate the concepts and properties of mixed fuzzy topological R-modules and bi-fuzzy topological R-modules.

Results and Discussion

3.1 bi-fuzzy top. R-modules and mixed fuzzy top. R-modules

We study mixed fuzzy top. R-module and fuzzynbhd of mixed fuzzy top. R-module

Definition 3.1.1

If M be an R-module with two fz. tops. R-module β and ρ. Then the triple (M, β, ρ) is known a bi- fz. top. R-module space.

Example 3.1.2

If M be an R-module with the indisc. fz. top. F_I and the disc. fz. Top. F_D. Then, $(M, F_I,\ F_D.)$ is a bi- fz. top. R-module

Theorem 3.1.3

If (M, β) and (M, ρ) be fz. tops. R-modules s.t $\rho \leq \beta$. Let ω_1, ω_2 be a fund. system of fz. nbhds of the zero element $0 \in M$ in the $\beta\ and\ \rho$ consecutively. Then

$\omega_1(\omega_2) = \{E \in J^M: \exists F \in \rho\ s.t\ cl_\beta(F) \leq E\}$ is a fund. system of fz. nbhds of 0

Proof

We clam the Theorem 1.7 justify by the class $\beta(\rho)$.

(**1**) Let $E \in \omega_1(\omega_2)$, then $\exists F \in \omega_2$ s.t $cl_\beta(F) \leq E$, by Theorem 1.7 (1)

$$-F \in \omega_2$$

Now

$cl_\beta(-F) = -cl_\beta(F) \leq -$E Also $-E(0) = E(-0) = E(0) > 0$

Thus

$$-E \in \omega_1(\omega_2)$$

(**2**) Let $E \in \omega_1(\omega_2)$Then $\exists F \in \omega_2$ by Theorem 1.7 (2) , $F + F \in \omega_2$. Now,

$$cl_\beta(F) + cl_\beta(F) \subseteq E + E$$

Also $(E + E)(0) = supmin\{E(0), E(0)\} = E(0) > 0$

Thus, $E + E \in \omega_1(\omega_2)$

(**3**) Let $E_1, E_2 \in \omega_1(\omega_2)$then $\exists\ F_1, F_2 \in \omega_2$ s.t $cl_\beta(F_1) \leq E_1$ and $cl_\beta(F_2) \leq E_2$. Since $F_1, F_2 \in \omega_2$ then by Theorem 1.7 (3) , we have $F_1 \wedge F_2 \in \omega_2$.

Now

$$cl_\beta(\ F_1 \wedge F_2) \leq cl_\beta(F_1) \wedge cl_\beta(F_2) \leq E_1 \wedge E_2$$

Thus,

$$E_1 \wedge E_2 \in \omega_1(\omega_2).\text{ Also } (E_1 \wedge E_2)(0) = \min\{E_1(0), E_2(0)\} > 0$$

(**4**) Let $E \in \omega_1(\omega_2)$ then there exists $F \in \omega_2$ and $W \in R$ with $cl_\beta(F) \leq E$, by Theorem 1.7 (4) , we have $W.F \leq E$.

Now

$$W.F(0) = \min\{W(0), F(0)\} > 0$$

Thus, $W.F \in \omega_1(\omega_2)$.

(5)

By the same way of (4) we can prove that $r.E \in \omega_1(\omega_2)$

(6)

Let $E \in \omega_1(\omega_2)$, $m \in M$ and F a Fz. nbhd of 0 in R, then by Theorem 1.7 (6) we get $F.m \leq E$. Now, $cl_\beta(F.m) = cl_\beta(F).m \leq E.m$. Also $(E.m)(0) = E(0) > 0$. Thus, $E.m \in \omega_1(\omega_2)$

Theorem 3.1.4

Let (M,β) and (M,ρ) be fz. tops. R-modules satisfy $\rho \leq \beta$ and ω_1, ω_2 be a fund. System of fz. nbhds of zero element $0 \in M$ in the fz. top. spaces β, ρ consecutively. $\omega_1(\omega_2) = \{E \in I^M: \exists F \in \rho\ s.t\ cl_\beta(F) \leq E\}$ is a fund. system of fz. nbhds of 0 justify theorem 2.3, then ∃! fz. top. $\beta(\rho)$ s.t $(M, \beta(\rho))$ is fz. top. R-module.

Proof

(1) Clearly $1_\emptyset$ and 1_M belong to $\beta(\rho)$
(2) Let $E_1, E_2 \in \beta(\rho)$ with $E_1(0) > 0$ and $E_2(0) > 0$ then $\exists$ $F_1, F_2 \in \omega_2$, with $F_1(0) > 0$ and $F_2(0) > 0$ s.t $F_1 \leq E_1, F_2 \leq E_2$ and $F_1 \wedge F_2 \in \rho$. Also $(F_1 \wedge F_2)(0) = \min\{F_1(0), F_2(0)\} > 0$ Now $cl_\beta(F_1 \wedge F_2) \leq cl_\beta(F_1) \wedge cl_\beta(F_2) \leq E_1 \wedge E$
Thus $E_1 \wedge E \in \beta(\rho)$
(iii)
Let I be an indexed and $\forall i \in I$ s.t $E_i \in \beta(\rho)$ and $E(0) > 0$. Then $\exists F_i \in \rho$ with $F_i(0) > 0$ s.t $cl_\beta(F_i) \leq E_i$.and $cl_\beta(F_i)(0) \geq F_i(0) > 0$. We get $cl_\beta(F) \leq E_i < \vee\{E_i\}$. Thus $\vee E_i \in \beta(\rho)$
(iv)
We claim the map $l: (R, \beta(\rho)_R) \times (M, \beta(\rho)_M) \longrightarrow (M, \beta(\rho)_M)$ defined by $l(r,m) = r.m$ $(r \in R, m \in M)$is a fz. conts. If $E_{r.m.}$be a fz. nbhd, then $\exists E \in \{E_m\}$ s.t $l^{-1}(E)(r,m) = E\big(l(r,m)\big) = E(r.m) > 0$. Since $E \in \{E_m\}$, by theorem 2.3 $\exists F \in E$ and a fz. nbhd W of r in R s.t $W.F \leq E$
Thus $l: R \times M \longrightarrow M$ is fz. Conts. Then $(M, \beta(\rho))$ is fz. top. R-module and clearly unique.

Definition 3.1.5

Let (M, β, ρ) be a bi- fz. top. R-module. The fz. Top. $\beta(\rho)$ specific on M by the class $\{E \in I^M: \exists F \in \beta\ s.t\ cl_\beta(F) \leq E\}$ of all fz. open nbhds of 0 s.t $(M, \beta(\rho))$ be a fz. top. R-module, is known as a mixed fz. top. R-module

Example 3.1.6

In the bi- fz. Top.R-module (M, β, ρ), let us put $\beta = F_I$, the inds. fz. top. on M and $\rho = F_D$, the disc. Fz. Top. on M, then, we have $\beta(\rho) = \beta$.

Theorem 3.1.7

Every fz. sub-R-module of mixed fz. top. R-module is a mixed fz. top. R-module.

Proof

Let E be a fz. subR-module of a bi- fuzzy top. R-module (M, β, ρ). clearly, $\beta_E$and ρ_E are relative fuzzy top. R-module on F, then $(F, \beta_E(\rho_E))$ where $\beta_E(\rho_E) = \{U \in J^E : \exists V \in \rho_E\ s.t\ cl_{\beta_E}(V) \leq U\}$, is fuzzy top. R-module space. Also, since $(M, \beta(\rho))$ is mixed fz. top. R-module, then$(\beta(\rho))_E$ is a relative mixed fz. top. R-module on E

Theorem 3.1.8

Let (M, β, ρ) be any bi- fuzzy top. R-module ring with $\beta(\rho)$ as the mixed fuzzy top. R-module on M. For any subring F of M, let $\beta/F, \rho/f$, and $\beta(\rho)/F$ be the fuzzy quotient top. R-module on M/F derived from the fuzzy top. R-module (M, β) , (M, ρ) and $(M, \beta(\rho))$ respectively. Then

$$\beta/F(\rho/F) = \beta(\rho)/F$$

Proof

By the same way of prove of theorem 3.1 [3]

Conclusion

This paper first introduces the concept "bi-fuzzy top. R-modules", which is the combined form of fuzzy topology and fuzzy algebra. It is similar to bi-fuzzy topological spaces as introduced by Lutfy et al.[12] in which bi-fuzzy topological space is a pair of fuzzy topological spaces that are equipped with fuzzy topology. This paper reports on the more generalized form of bi-fuzzy top. R-modules' definition that includes bi-fuzzy topological spaces as a special case. In this paper, the properties of bi-fuzzy top. R-modules, fuzzy neighborhoods, fuzzy submodules, and fuzzy quotient modules are analyzed.The results obtained are consistent with the earlier findings on fuzzy topological spaces and fuzzy algebraic structures too. An example of such works could be Basim et al. [2] considering the properties of fuzzy topological spaces such as fuzzy topology, fuzzy convergence and fuzzy subspaces. Another example would be deb et al. [8] considering the properties of the fuzzy algebraic structures such as fuzzy submodules and fuzzy quotient modules.

The research constructs mixed fuzzy top. R-modules by applying the results on fuzzy neighborhoods to bi-fuzzy top. R-modules. This construction is similar to the construction of mixed fuzzy topological spaces presented by Chang [1]. In Chang's construction, two fuzzy topological spaces with distinct fuzzy topologies are put together to create a new mixed fuzzy topological space. If L be a set and S is a fuzzy topology in X: indicated in the research finding every fuzzy subR-module of the mixed fuzzy top. R-module is a mixed fuzzy top. R-module. This is the same result as the one in the study by Mohammed et al. [10] who showed that every fuzzy submodule of a fuzzy algebraic structure is a fuzzy algebraic structure. The paper shows that for any bi- fuzzy top. R-module ring with a mixed fuzzy top. R-module on, and for any subring of , the fuzzy quotient top. R-module on derived from the fuzzy top. R-module is a mixed fuzzy top. R-module. This result is similar to the result presented by Basim et al. [3], who showed that for any fuzzy algebraic structure, the fuzzy quotient module derived from the fuzzy submodule is a fuzzy algebraic structure.

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